WEIGHTS IN McMAHON PAIRING - REVISED

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May 2015

Contents

References

[1] Geoff Kaniuk,Weights in McMahon Pairing, January 2012,
www.kaniuk.co.uk/pairing/mcmahon-weights.pdf

1 PURPOSE

It is the intention of this article to:

- Revise and summarise the weight model presented in Ref [1].
- Complete detail that was previously left open.
- Reduce the number of parameters needed to specify the rule deviations.
- Correct errors in the model presentation.

This document forms the basis of the calculation of weights for the pairing in GoDraw.

2 THE WEIGHT MODEL

2.1 Even, Uneven, and Handicap games

Given a pairing between players i and j with McMahon scores m_i, m_j, the game is classified as follows:

$$\begin{aligned} \text{even} : |m_i - m_j| &< |H| \\ \text{uneven} : |m_i - m_j| &= |H| \\ \text{handicap} : |m_i - m_j| &> |H| \end{aligned}$$

The non-zero quantity H is the *handicap-offset*, and is often set to -1. The handicap awarded to the weaker player m_i in a handicap game is usually $m_j - m_i + H$.

The rule "minimise the McMahon score difference between players" automatically ensures that players receive the minimum possible handicap when needed. In some cases this could lead to pairings with handicaps larger than 9 stones, and this is sometimes unavoidable at the bottom end of the draw. At the top end of the draw it is normally unacceptable to have any handicap games, and so the pairing system excludes such pairings explicitly rather than relying on weight values.

2.2 Weight Components

The total weight for the pairing ij between any two players i and j is a sum of components:

$$W_{ij} = \sum_{k=1\cdots 8} w_{ij}^k W_k$$

W_k is the weight coefficient for the component k and may depend on the players' grades, but not on any other player details. w_{ij}^k is the weight component for the pairing. It depends only on player performance and history as determined by the tournament, and is independent of player grades.

The components in the sum are identified by index k and correspond in order to the rules:

1. **mms**: minimise the McMahon score difference.
2. **even:** rules for pairing within a McMahon group.
3. **uneven:** rules for pairing across distinct McMahon groups.
4. **area:** prevent same-area games.
5. **colour:** minimise player colour imbalance.
6. **banding:** pair players within bands defined by grade.
7. **play-in:** players in a group prefer to play each other.
8. **play-out:** players in a group prefer not to play each other.

For each component k there is defined a rule-deviation δ_{ij}^k dependent on player performance. The weight component for the pairing ij is then given by:

$$w_{ij}^k = \operatorname{sech}(\lambda_k \delta_{ij}^k) \tag{1}$$

The $\operatorname{sech}(x)$ function ensures that the optimal weight is given to the pairing which best satisfies the rules i.e. $\delta_{ij}^k = 0$, and smaller weights are given to pairings with larger rule deviations. Furthermore, the non-linearity of $\operatorname{sech}(x)$ removes the degeneracy inherent in summing weights which are linear functions of the deviation.

Finally, note that weights are defined for players i and j only if a potential pairing of the players exists. In particular there is no potential pairing if they have played each other in a previous round.

2.3 Player details

The player details required for evaluation of weights is listed in the following table. Here and in subsequent sections, McMahon is abbreviated by MM:

NAME	SYMBOL	INFORMAL DEFINITION
grade	g_i	entry grade of the player.
mmi	m_i^o	initial MM score from grade & bar.
mms	m_i	current MM score.
seed	s_i	used for pairing in MM groups
drawn up	U_i	number of games drawn up.
drawn down	D_i	number of games drawn down.
drawn up losses	U_i^-	number of games drawn up and lost.
drawn down wins	D_i^+	number of games drawn down and won.
mm balance	m_i^{bal}	the difference $U_i - D_i$ or $U_i^- - D_i^+$
mm uneven	m_i^{out}	the sum $U_i + D_i$ or $U_i^- + D_i^+$
colour	c_i^r	round r: +1(W), -1(B), 0(no play).
colour balance	C_i^r	total games W - games B at round r.
area count	A	distinct areas in a MM group.
area seed	a_i	from mixed-area-algorithm.
band	b_i	player's band index.
play in	P_i^{in}	played in the group.
play out	P_i^{out}	played out of the group.

3 ELEMENTARY RULE DEVIATIONS

3.1 mms

The McMahon rule-deviation is

$$\delta_{ij}^{mms} = m_i - m_j$$

This component of weight is applied to every pairing without exception.

3.2 banding

There are K bands indexed by k, and a player i has a band index b_i lying in the range $[1, K]$. The rule-deviation is:

$$\delta_{ij}^{band} = b_i - b_j$$

The rule is applied if the banding feature has been selected.

4 EVEN GAMES

This section considers the rule deviations for pairing players i and j when the player's McMahon scores are equal. We normalise player seeds within a MM group to lie in the range $0 \leq s \leq 1$. The pairing is governed by one of the following five strategies, each of which provides an alternative weight component:

4.1 Random

All players in the McMahon group get the same weight component. The rule-deviation is

$$\delta_{ij}^{random} = 0$$

Randomness is obtained by a random shuffle of each MM group prior to pairing.

4.2 Fine grain

The player seed provides a finer grained estimate of strength than pure MM score:

$$\delta_{ij}^{\text{fine-grain}} = s_i - s_j$$

Closely seeded players get the highest weights.

4.3 Split and fold

The conventional strategy of folding the top half of the group onto the bottom half can be expressed by the rule-deviation:

$$\delta_{ij}^{\text{split-fold}} = |s_i + s_j - 1|$$

In this form, a pairing of two high seed players may have a similar deviation to two low seed players. Since the point of split and fold is to maximise the chances of the stronger players winning as many games as possible, we require that a high seed pairing should get a lower weight than a low seed pairing. A rule-deviation which meets this requirement is

$$\delta_{ij}^{\text{split-fold-ex}} = |s_i + s_j - 1|\{1 + \tfrac{1}{2}(s_i + s_j)\}$$

This significantly increases the deviation for a high seed pair, but does not change the deviation much for a low seed pair.

4.4 Split and match

In the conventional split and match strategy, the top half of the group slides onto the bottom half of the group, and this can be expressed by the rule-deviation:

$$\delta_{ij}^{\text{split-match}} = ||s_i - s_j| - \tfrac{1}{2}|$$

The ideal pairing has a seed difference of $\frac{1}{2}$, but *any* constant seed difference gives the same deviation. So again we modify the rule-deviation to give larger deviations for a pairing of higher seeded players:

$$\delta_{ij}^{\text{split-match-ex}} = ||s_i - s_j| - \tfrac{1}{2}|\{1 + \tfrac{1}{2}(s_i + s_j)\}$$

4.5 Split and mix

The two halves are paired at random. Any player i with seed $0 \leq s_i \leq \frac{1}{2}$ and any player j with seed $\frac{1}{2} \leq s_j \leq 1$ form an ideal pairing. This condition can be captured by the expression:

$$\Delta_{ij} = (s_i - \tfrac{1}{2})(s_j - \tfrac{1}{2}) \leq 0$$

Hence we can define a rule-deviation by:

$$\begin{aligned} \delta_{ij}^{\text{split-mix}} &= 0, & \text{if } \Delta_{ij} \leq 0 \\ \delta_{ij}^{\text{split-mix}} &= \Delta_{ij}\{1 + \tfrac{1}{2}(s_i + s_j)\}, & \text{otherwise} \end{aligned}$$

5 UNEVEN GAMES

The rule-deviations for pairing across groups with different McMahon scores are applied only to non-handicap games.

For a given player, denote the number of games played up by U, and the number played down by D. The pairing for uneven games aims to satisfy two requirements:

- Minimise the number of uneven games $m^{out} = U + D$.
- Minimise the difference $m^{bal} = U - D$.

In the ideal uneven pairing, each player has played just one uneven game and we try to reduce the balances of both players simultaneously. In order to minimise the number of uneven games, we firstly aim to allow uneven games only between players from *neighbouring* McMahon groups γ_i and γ_j. To this end define a group deviation γ_{ij} by:

$$\gamma_{ij} = ||\gamma_i - \gamma_j| - 1|$$

Then $\gamma_{ij} = 0$ when the indexes to the McMahon groups differ by 1 in magnitude.

We can now define a rule deviation via:

$$\delta_{ij}^{out} = \gamma_{ij} + m_i^{out} + m_j^{out} - 2$$

The rule-deviation is zero in the ideal case, and increases in magnitude as group distance or the number of uneven games played increases .

Next we consider two strategies for reducing the balance m^{bal}:

full a game played up is compensated by a game played down in a later round, irrespective of the game result.

partial compensation is applied as in *full* only if the player perform as expected i.e. plays up and loses or plays down and wins.

5.1 Full balance

In the full balance rule, priority is given to reducing the balance to zero, even though this may mean increasing the number of uneven games. We can assess the *quality* of the pairing as:

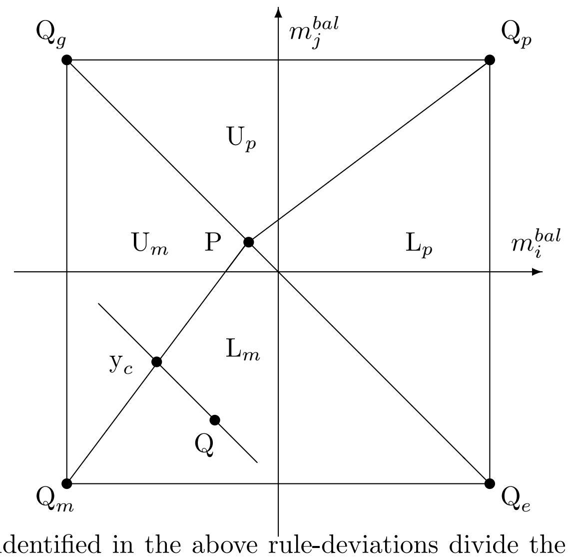
poor The balance gets worse for both players.

fair The balance reduces for one player, but increases for the other.

good The balance reduces for both players.

The range of m^{bal} for any player is $[-r, r]$, where r is the number of rounds so far played. In what follows, assume that $m_i < m_j$, i.e. i plays up to j. For the ideal pairing $m_i^{bal} = -1$ (so i *wants* to play up), $m_j^{bal} = +1$ (j *wants* to play down). We can assign a value to the rule-deviation for particular combinations of m_i^{bal}, m_j^{bal} as follows:

- $\delta = 0$ for the *perfect* case shown at the point P $= (-1, 1)$ in the diagram.
- $\delta = 0$ along the line from P to $\mathrm{Q}_g = (-r, r)$. These are *good* cases because i plays up as desired and j gets to play down as desired.
- δ increases from 0 to $2r$ along the line from P to $\mathrm{Q}_e = (r, -r)$. These are *poor* quality pairings because for these points, i wants to play *down* and j wants to play up, but i is playing *up* by our assumption $m_i < m_j$.
- δ increases from 0 to r along either of the lines joining P to $\mathrm{Q}_p = (r, r)$ or $\mathrm{Q}_m = (-r, -r)$. These are *fair* pairings, because in most cases the balances have the same signs. So the balance for one of the players improves, but for the other it will worsen.



The four lines identified in the above rule-deviations divide the balance space (m_i^{bal}, m_j^{bal}) into 4 triangular regions: $\mathrm{U}_m, \mathrm{U}_p, \mathrm{L}_m, \mathrm{L}_p$, sharing the common vertex P. A value for the rule-deviation δ_{ij}^{bal} can be obtained for any point Q lying in one of these regions by linear interpolation from deviation values specified at the vertices of the triangle containing the point Q.

Identification of the particular region containing a given point (m_i^{bal}, m_j^{bal}) is more easily achieved by considering transformed co-ordinates:

$$x = \tfrac{1}{2}(m_j^{bal} + m_i^{bal})$$
$$y = \tfrac{1}{2}(m_j^{bal} - m_i^{bal})$$

If $x \leq 0$ then $Q = (x, y)$ is in either of U_m or L_m. The line through Q with constant x co-ordinate meets the line $\mathrm{Q}_m\mathrm{P}$ in the critical point $y_c = (r + x)/r$. So if $y \geq y_c$, Q lies in U_m, otherwise it lies in L_m.

The same argument allows us to identify the location of Q in U_p or L_p when $x > 0$, but here the critical value is $y_c = (r - x)/r$. We can combine the expressions for the critical value into the single form $y_c = (r - |x|)/r$

Once Q is located, the value of the deviation is obtained by linear interpolation as detailed in Appendix A:

$$\begin{aligned} \mathrm{U}_m \text{ or } \mathrm{U}_p: \ & \delta = |x| \\ \mathrm{L}_m \text{ or } \mathrm{L}_p: \ & \delta = [(r-1)|x| + 2r(1-y)]/(r+1) \end{aligned} \tag{2}$$

5.2 Partial

Priority is given to minimising the number of uneven games at the possible expense of an increased balance. If the player performs unexpectedly i.e. plays up and wins or plays down and loses, we do not try to later reduce the balance. This helps to give players more equal opponents. Split the played up and played down counts into winning and losing parts: U^-, U^+, D^-, D^+ where $+$ counts wins and $-$ counts losses. We then define a partial balance value:

$$\hat{m}^{bal} = U^- - D^+$$

Now the range of U^- and D^+ is the same as the range of U and D i.e. $[0, r]$, so $\hat{m}^{bal}$ has the same range as m^{bal} namely $[-r, r]$. We can apply the same pairing assessment to $\hat{m}^{bal}$ as we did to m^{bal} to arrive at a rule-deviation $\hat{\delta}_{ij}^{bal}$ which has exactly the same form as the rule-deviation specified for the full rule in Equation (2).

5.3 Seeding

We can utilise a seed for pairing across different McMahon groups. Suppose the lower group $\mathcal{L}$ has players with seeds l_i, and the upper group $\mathcal{U}$ has players with seeds u_j. In $\mathcal{L}$, the seeds run from 0 to L, and for $\mathcal{U}$ the seeds run from 0 to U. In either group the weakest player has seed 0. We consider three pairing strategies:

fold The weakest in $\mathcal{L}$ plays the strongest in $\mathcal{U}$.

match The strongest in $\mathcal{L}$ plays the strongest in $\mathcal{U}$.

nearest The strongest in $\mathcal{L}$ plays the weakest in $\mathcal{U}$.

Rule-deviations expressed in terms of seed values are:

$$\begin{aligned} \delta_{ij}^{fold} &= l_i + U - u_j \\ \delta_{ij}^{match} &= L - l_i + U - u_j \\ \delta_{ij}^{nearest} &= L - l_i + u_j \end{aligned}$$

For each pairing strategy, the condition that the seeds are positive forces the deviation to lie in the range $[0, L + U]$. For each one there is a unique point where the deviation is zero.

5.4 Weight component for uneven games

We have established three kinds of rule-deviation for assessing the pairing when players lie in neighbouring McMahon groups:

$$\delta_{ij}^{out} \quad \delta_{ij}^{ubal} \quad \delta_{ij}^{seed}$$

Here *seed* is one of $fold$, $match$, or $nearest$ and *ubal* is either the full balance rule *bal* or the partial rule $\hat{bal}$.

The weight components associated with these deviations are:

$$w_{ij}^{out} = sech(\lambda_{out}\delta_{ij}^{out})$$
$$w_{ij}^{ubal} = sech(\lambda_{ubal}\delta_{ij}^{ubal})$$
$$w_{ij}^{seed} = sech(\lambda_{seed}\delta_{ij}^{seed})$$

The complete weight component for uneven pairing a function of the individual components associated with each deviation:

$$w_{ij}^{uneven} = \Omega(w_{ij}^{out}, w_{ij}^{ubal}, w_{ij}^{seed})$$

where Ω has the range $[0, 1]$.

6 AREA PAIRING

The need to keep players from the same area apart arises both in pairing within a McMahon group and in pairing across different McMahon groups. We generate an area seed by first sorting the group of size $2n$ in decreasing area size. Then split the group in two parts - say A and B. Part A can be indexed from $[1 \cdots n]$, and part B can be indexed from $[-1 \cdots -n]$.

The indexes provide a unique *area seed* a_i for each player in the group. This can be used as the area rule deviation:

$$\delta_{ij}^{area} = a_i + a_j$$

Players matched with low rule deviation are selected from different areas.

6.1 Mixed area pairing

A possibly better method starts by setting the area seed of every player in the draw to 0. Then prior to pairing, apply the following algorithm to each MM group below the bar:

MA0. Initialise:
Set variable a to 1. Random shuffle the MM group.

MA1. Sort in order of decreasing area size:
Order the group by *area* (country, club) and arrange in subsets of constant area with decreasing area size. We obtain a sequence $\{A_i : i = 1 \cdots n\}$, in which each set A_i consists of players from the same area.

MA2. Seed:
Set the area seed of the player at the front of A_1 to +a. Set the area seed of the player at the back of A_n to -a. Remove these two seeded players from their respective area sets.

MA3. A_1 status test:
If A_1 has less than 2 players, or if it is the only set, halt.

MA4. Next seed:
If A_1 is no longer the largest set, increase a by 1. Continue at MA1.

Each player i in the group now has an area seed, and any pair of players with equal and opposite seeds are from different areas.

If there is one dominant area in the group whose size is larger than half the size of the group, then there will be some players left in the group with seed 0, and they are likely to get paired as expected.

7 COLOUR

A player's colour in round r is denoted by $C(r)$ and takes the values $+1$ for white, -1 for black, or 0 if player was absent in round r. The *colour balance* for a player is the excess of games played as white over those played as black and is given by:

$$C^{bal}(r) = \sum_{s=1}^{r} C(s)$$

If the colour balances for two players are equal and opposite, the opportunity exists to *both* reduce the balance *and* alternate the colours. We define a measure of a player's alternation pattern by :

$$C^{alt}(r) = \sum_{s=2}^{r} |C(s) - C(s-1)|/2$$

Thus during pairing, or assignment of colours after pairing, we consider the change D^{bal} in the magnitude of the colour balance and the change D^{alt} in the alternation induced by a particular colour choice:

$$D^{bal}(r) = |C^{bal}(r)| - |C^{bal}(r-1)| \quad (3)$$
$$D^{alt}(r) = C^{alt}(r) - C^{alt}(r-1) \quad (4)$$
$$\quad (5)$$

A players colour balance can either increase or decrease by 1 at each new round, but the alternation either stays the same or increases by 1. This implies that D^{bal} and D^{alt} have the properties:

$$\begin{aligned} C^{bal}(r-1) = 0 &\Rightarrow |D^{bal}(r)| = 1 \\ C(r) = \operatorname{sign}(C^{bal}(r-1)) &\Rightarrow D^{bal}(r) = 1 \\ C(r) = -\operatorname{sign}(C^{bal}(r-1)) &\Rightarrow D^{bal}(r) = -1 \\ C(r) = C(r-1) &\Rightarrow D^{alt}(r) = 0 \\ C(r) = -C(r-1) &\Rightarrow D^{alt}(r) = 1 \end{aligned} \quad (6)$$

Suppose now players i and j are paired at round r. They must have opposing colours i.e.

$$C_i(r) + C_j(r) = 0$$

The choice of colours induces changes in both players' values for D^{bal} and D^{alt}. We consider the sum of the changes:

$$X_{ij} = D_i^{bal}(r) + D_j^{bal}(r) \quad (7)$$

$$Y_{ij} = D_i^{alt}(r) + D_j^{alt}(r) \quad (8)$$

$$(9)$$

Since $D^{bal} = \pm 1$, X_{ij} can take on values -2,0,2, and since D^{alt} is 0 or 1 it follows that Y_{ij} can take on values 0,1,2. We encode the values of X_{ij} and Y_{ij} into *colour index* C_{ij} for the pairing via the transforms:

$$C_{ij} = 3(X_{ij}/2 + 1) + 2 - Y_{ij}$$

$$X_{ij} = 2(\lfloor C_{ij}/3 \rfloor - 1) \quad (10)$$

$$Y_{ij} = 2 - C_{ij} + 3(X_{ij}/2 + 1) \quad (11)$$

$$(12)$$

From the point of view of colour assignment,we obtain the best pairing when the balances for*both* i and j reduce in magnitude *and* the alternation for each increases. In this case the colour index has its lowest value of 0. We obtain the worst pairing when the balances for *both* i and j increase in magnitude *and* the alternation of each stays the same. In this case the the colour index has its largest value of 8.

The mid value for the colour index is 4. From Equation (10) $X_{ij} = 0$ So by Equation (7), one players colour balance improves, but the other player has a worse balance. From Equation (11), $Y_{ij} = 1$, so from Equation (8) one players alternation stays the same and the others improves.

For the purpose of weight assignment we can define a rule deviation for rounds from 2 on:

$$\delta_{ij}^{colour} = (1 + C_{ij})/\Gamma_{ij} \quad (13)$$

$$\Gamma_{ij} = (1 + |C_i^{bal}| + |C_j^{bal}| + |C_i^{alt}| + |C_i^{alt}|)/(4r - 1) \quad (14)$$

$$(15)$$

The largest balance value is r and the largest alternation value is $r - 1$. The factor Γ_{ij} ensures that players with the poorest balances and alternations get the best weights for improvement.

BALANCES C_i, C_j	COLOURS c_i, c_j	DEVIATION $\lvert\delta^{colour}\rvert$
$C_i + C_j = 0$	opposite	0
$C_i + C_j = 0$	same	$\lvert\delta^{colour}\rvert \leq 1$
$C_i + C_j \neq 0$	any	$1 \leq \lvert\delta^{colour}\rvert \leq 2r$

The colour weight is applied to non-handicap games.

8 PLAY-IN

Each player in a *play-in* group, is given a count P^{in} of the number of games played against another player *in* the group up to the last completed round r. If a given MM group contains one or more such play-in groups, we give each such group an index q. Each player i in the group q is given a group index $i_G = q$. Players in the MM group not in any of the play-in groups are given a group index 0, and a play-in count of r. We define a play-in rule-deviation as follows:

$$\delta_{ij}^{play-in} = 0 \qquad \text{if } i_G = j_G$$

$$\delta_{ij}^{play-in} = \frac{2r + 1 - P_i^{in} - P_j^{in}}{2r + 1} \qquad \text{if } i_G \neq j_G$$

Players i, j in different groups with zero play-in counts get the maximum deviation of 1. Players in different groups who have played all rounds *in* will have play-in counts of r. Such pairings would get a small non-zero deviation of $\frac{1}{2r+1}$, and so may be chosen to play *out* if needed.

9 PLAY-OUT

As with the play-in groups, each player in a *play-out* group, is given a count P^{out} recording the number of games played *outside* the group. Again we label all the play-out groups in a MM group with an index q, just as in the previous section.

$$\delta_{ij}^{play-out} = 0 \qquad \text{if } i_G \neq j_G$$

$$\delta_{ij}^{play-out} = \frac{2r + 1 - P_i^{out} - P_j^{out}}{2r + 1} \qquad \text{if } i_G = j_G$$

Two players in the same group attract the maximum deviation if their play-out counts are zero. If however their play-out counts are maximal i.e. have the value r, then the rule-deviation for the pairing drops to $\frac{1}{2r+1}$ and they may be chosen to play *in*.

10 PAIRING QUALITY

It is useful to monitor the performance of the pairing algorithm by assessing how well it achieves its aim in meeting the requirement of the McMahon rules. This means monitoring at the very least:

- Games with unequal McMahon scores.
- Same-area games.
- Colour balance and alternation.

The following sections present methods for estimating the pairing quality for the three identified features.

10.1 Uneven

At the beginning of each round, players in the draw are sorted into MM groups. The pattern of parities in the ensuing group sizes allows us to estimate the number of games with differing MM scores. Summing quantities over all rounds, we define the *McMahon-Quality* as:

$$
\begin{aligned}
Q_{mms} &= 1 - |M_{mms} - E_{mms}|/N_{mmg} \\
M_{mms} &- \text{ actual number of unequal games} \\
E_{mms} &- \text{ expected number of unequal games} \\
N_{game} &- \text{ total number of games}
\end{aligned}
$$

10.2 Area

Each MM group is subdivided into groups of the same area. Suppose a MM group has size M, and the largest contained area group has size A.

Split the MM group in two, and slide the two halves to match. It is apparent that if $A \leq \frac{1}{2}M$ then there should be no same-area games. Otherwise the number of same-area games is the portion of A that got cut in the split, i.e. $A - \frac{1}{2}M$. We can estimate the number of same-country games and the number of same-club games separately for each round, and summing over all rounds we define the *Area-Quality* for games *below the bar* by:

$$
\begin{aligned}
Q_{area} &= 1 - (\Delta_{country} + \Delta_{club})/N_{gameb} \\
\Delta_{region} &= |M_{region} - E_{region}| \\
M_{region} &- \text{ actual number of same-region games} \\
E_{region} &- \text{ expected number of same-region games} \\
N_{gameb} &- \text{ number of games below the bar}
\end{aligned}
$$

Here *region* is *country* or *club*.

10.3 Colour

At the end of the tournament each player has acquired a colour balance with magnitude $|C^{bal}|$. The possible range of $|C^{bal}|$ depends on whether the tournament has an even or odd number of rounds. Define E^{bal} to be 0 if r is even and 1 if r is odd. Then

$$E^{bal} \leq |C^{bal}| \leq r$$

Each player also acquires an alternation value in the range $[0 \cdots r-1]$ and we can define a combined *colour value* C_i^{val} for each player in a tournament with r rounds by

$$C_i^{val} = (|C^{bal}| - E^{bal} + r - 1 - C^{alt}/(2r - 1 - E^{bal})$$

The colour value for each player then lies in the range $0 \leq C_i^{val} \leq 1$. The best value is 0 and the worst is 1. We can therefore define a colour quality for the whole tournament also in the same range by:

$$Q_{colour} = \sqrt{1/n \sum_{i=1 \cdots n} (C_i^{val})^2}$$

The summation is over all players and so includes handicap games if any.

11 GRADE DEPENDENT COEFFICIENTS

In the previous sections, rule deviations have been defined for each of the 8 weight components. For uneven games the weight component has been split into 3 sub-components. In all cases:

- The rule-deviation is fully determined by the tournament parameters and the players' performance.
- The weight component is expressed as a function of the rule-deviation by Equation (1) i.e. $w^k = \text{sech}(\lambda_k\, \delta^k)$.

The factor λ_k is a shape factor: it determines how quickly a component weight reduces as the rule-deviation increases. This in turn determines the *severity* with which a rule is applied. The larger it is, the more quickly does the weight reduce, and this prunes the pairings with hight weight. Clearly not all rules should be applied with equal severity across all grades in the draw.

The following lists the general behaviour of the core McMahon rules across all grades. The components *mms, even, uneven* are all associated with rule deviations that depend on McMahon score (we class these as *scoring components*) so their behaviour will have the same characteristics.

scoring component We require good rule conformance at the strong end of the draw, but it can be relaxed at the weaker end.

area component Mixed-area pairing should be thorough at the weak end of the draw, but is not so important at the strong end of the draw.

colour component Colour balance minimisation is not achieved at the extreme ends of the draw, but is feasible in the mid-ranges of the draw.

The above behaviour is well captured by the following forms:

$$\begin{aligned} \lambda_{score}(g) &= \tfrac{1}{2}(1 + \tanh(\gamma_{score}(g - g_{score}))), & g_{score} &\approx 1\text{kyu} \\ \lambda_{area}(g) &= \tfrac{1}{2}(1 - \tanh(\gamma_{area}(g - g_{area}))), & g_{area} &\approx 8\text{kyu} \\ \lambda_{colour}(g) &= sech(\gamma_{colour}(g - g_{colour})), & g_{colour} &\approx 3\text{kyu} \end{aligned}$$

A UNEVEN RULE DEVIATION

We specify δ at 5 points. In each of the 4 triangular regions identified in Figure 1, the rule-deviation is given by the linear form:

$$\delta = a + bx + cy$$

All regions have the point $\mathrm{P} = (0, 1)$ in common, and at this point $\delta = 0$. This immediately gives the specialisation $a + c = 0$, so the form for delta reduces to:

$$\delta = bx + c(y - 1) \tag{16}$$

A.1 Critical point y_c

In the transformed representation, the points $\mathrm{P}, \mathrm{Q}_m, \mathrm{Q}_p$ have the (x, y) co-ordinates:

$$\mathrm{P} = (0, 1) \qquad \mathrm{Q}_m = (-r, 0) \qquad \mathrm{Q}_p = (r, 0)$$

The line $\mathrm{Q}_m\mathrm{P}$ has slope $\frac{1}{r}$, intercept 1 on the Y-axis, and so has equation $y = \frac{1}{r}x + 1$.

The line PQ_p has slope $-\frac{1}{r}$, intercept 1 on the Y-axis, and so has equation $y = -\frac{1}{r}x + 1$.

A.2 Regions $\mathbf{U}_m$, $\mathbf{U}_p$

These regions have the point $\mathrm{Q}_g = (0, r)$ in common, where again $\delta = 0$. This gives $0 = c(r - 1)$ i.e. $c = 0$, leaving (16) simplified to $\delta = bx$ in these regions.

$$\begin{array}{ll} \text{At } \mathrm{Q}_m = (-r, 0), & \delta = r = -br \;\text{ i.e. } b = -1 \\ \text{At } \mathrm{Q}_p = (r, 0), & \delta = r = br \quad\;\text{ i.e. } b = 1 \end{array}$$

The two resulting expression for δ can be combined into the single statement:

$$\delta = |x|$$

for any point in U_m or U_p.

A.3 Regions $\mathbf{L}_m$, $\mathbf{L}_p$

Apart from P, the common point for these two regions is $\mathrm{Q}_b = (r, -r)$, and there we have specified $\delta = 2r$. Equation (16) then furnishes the relation $2r = -c(r + 1)$, and the form reduces to

$$\delta = bx + 2r(1 - y)/(1 + r)$$

Hence at the point $Q_m = (-r, 0)$, where $\delta = r$ we get:

$$r = -br + 2r/(1 + r)$$
$$\text{i.e. } b = -(r - 1)/(r + 1)$$

Again $\delta = r$ at Q_p and we see that:

$$r = br + 2r/(1 + r)$$
$$\text{i.e. } b = (r - 1)/(r + 1)$$

Finally, we can combine the two resulting expressions for delta to give the single form:

$$\delta = (r - 1)/(r + 1)|x| + 2r(1 - y)/(r + 1)$$

for any point in L_m or L_p.